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STRUCTURAL DIAGRAM AND PROPERTIES OF A COHERENT ANALYZER

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ANNOTATION

The basis of the coherent analyzer is a multiphase synchronous detector. The big advantage of the coherent analyzer, as compared to all existing spectrum analyzers, is the possiblity of determining the phase separated from the frequency component. Since any time function can be expanded into two spectral functions, it is possible to considerably reduce the volume of stored information without loss of time signal quality. This means that the equipment used to reproduce time signals can be changed substantially. This paper, in addition, reviews questions associated with the accuracy of determining the amplitude and phase of the frequency components of an input signal.

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ABSTRACT. The development of a coherent analyzer is discussed, as are questions associated with the accuracy of determining the amplitude and phase of the frequency components of an input signal.

Introduction /3*

The rapid development of radioelectronics, the deep involvement of the latter in science and engineering, and the creation and use of what are, in principle, new radioelectronic systems, are closely associated with the development of many new measuring devices and measuring methods.

Radioelectronic measuring techniques have undergone significant changes in the past decade. New instruments have appeared, measurement accuracy has increased, the speed and reliability of measurements have increased, the limits of parameters measured have been expanded, and equipment for measuring and investigating magnitudes and characteristics never before measured has been built.

The spectral method of making analyses has been most widely expanded. Spectral method of making investigations are widely used to study a variety of physical phenomena. The method is based on the representation of the unknown time process in the form of a sum of the harmonic components. The results of measurements made using the spectral method are convenient for review purposes, as well as for subsequent computer processing.

Experimental study of the characteristics of random processes plays a significant role in solving problems of detection and separation of signals from noise. Moreover, study of the spectral composition of random signals plays an important role in radioastronomy. The spectral composition of reflected, or direct, radio radiation from the planets in the solar system is used to refine the compositions of their atmospheres, surface temperatures, rate of rotation about their axes, /4
and many other parameters.

The frequency range encompassed by spectrum analyzers is very broad, from a

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* Numbers in the margin indicate pagination in the foreign text.

few Hz to hundreds of GHz. Devices using relatively high frequencies have acceptable size and weight, as well as technical, characteristics, but those for use in the sonic and subsonic frequency ranges are not quite as good. Here the basic components, the resonators, are very cumbersome and heavy, and one could wish for better technical characteristics [1-4, 6]. Recourse is being taken to a variety of auxiliary devices, to the method with magnetic recording [5, 7], for example, in order to somehow improve the characteristics of analyzers in the subsonic frequency band.

In addition, the synchronous method can be used to separate the harmonic components in the low and extremely-low frequency regions. The principle of synchronous detection has been described in many articles. Monograph [8] contains a wide-ranging list of sources on the subject.

A significant drawback of the synchronous method is the need to synchronize the frequency of the internal oscillator. The method that synchronizes the internal oscillator and the input signal can be used only to separate one frequency component. The drawback of synchronous detection, involving the need to synchronize the internal oscillator, becomes substantial if frequency analysis of an unknown signal is required.

Several signals, displaced with respect to phase, can be used to overcome the influence of phase in a synchronous detector.

An unknown signal spectrum analyzer can be built using a multiphase synchronous detector. This analyzer also makes it possible to construct the phase-frequency characteristic curve of the signal analyzed.

Despite the possibilities, the scientific and technical literature does not /5 yet contain any information on a multiphase coherent analyzer. No presently available analyzers can determine the phase-frequency characteristics of an unknown signal.

1. Possible Principles of Operation of a Multiphase Coherent Analyzer /6

Synchronous and asynchronous methods of separating a useful signal are used more and more widely today in measurement engineering, automation, and radio engineering.

The process of multiplying an unknown input fraction, x(t), by a periodic internal function, $y(t)$,

$$z(t) = c \cdot x(t) \cdot y(t) \tag{1.1}$$

is involved in the synchronous method of separating the frequency components of a signal.

The internal periodic function can be harmonic, as well as relay. Signals of the relay type have been most widely used because they are easier to realize than the harmonic type.

If these functions are represented in the form

$$x(t) = \sum_{i=0,1,2\dots}^{n} x_i \sin(i\omega t + \varphi), \text{ and } y(t) = \sum_{m=1,3,5\dots}^{n} y_m \sin m\omega t,$$

then

$$z(t) = c \sum_{i=0,1,2\dots}^{n} x_i \sin(i\omega t + \varphi) \cdot \sum_{m=1,3,5\dots}^{n} y_m \sin m\omega t.$$

Multiplying these functions, we obtain amplitudes of harmonics containing frequencies $(i\omega \pm m\omega)$. When $i\omega = m\omega$ we have the constant component

$$z_0 = c \sum_{i=m=1,3,5\dots}^{n} \frac{x_i y_m}{2} \cos \varphi, \tag{1.2}$$

This indicates that when there is coincidence of the internal function with the external harmonic, the magnitude of the constant component depends on the phase shift between these frequencies. If the function z_0 now is fed into an RC filter with a transfer coefficient of

$$K_t = \frac{1}{\sqrt{1+T^2\ (i\omega \pm m\omega)^2}}, \quad \text{when } m = 1,\ K_t = \frac{1}{\sqrt{i + \omega^2 T^2}}, \tag{1.3}$$

it will significantly weaken the frequency components of the result of the multiplication and will separate the constant component characterizing the equality of the frequencies. /7

Thus, in order to determine the frequency components of the input function it is necessary to multiply the function by the corresponding harmonic. We can, by changing the frequency of the internal oscillator, separate the amplitudes of all frequency components of the input signal.

But we encounter difficulties in practice that are associated with the phase coefficient, cos φ, which will introduce a substantial correction in the operation of the device when separating the different frequency components. Even if the frequency of the separated harmonic were to coincide with the frequency of the internal oscillator the signal would be zero at the output if there were a shift in phase between them of 90°. The phase will change constantly if the frequencies are unstable with respect to each other, with the output amplitude of the signal changing from maximum to zero. Beat frequencies will be generated. These can be very low frequency, and no filters of any kind will be able to smooth out the pulsation in the output signal. The most acceptable solution is to use a multiphase system.

Figure 1 shows a block diagram of a multiphase analyzer.

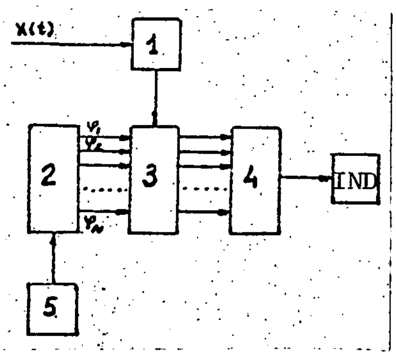

Figure 1.

The signal to be studied is supplied to /8 the input device, 1. This device should provide the required passband and dynamic voltage range. The signal flows from the output of 1 to the multiplier, 3, which is supplied with a signal from the internal oscillator by another input. Device 3 multiplies the two signals and the result is fed into an RC filter. The output signal from the multiplier is the output from the RC filter. The number of outputs is determined by the number of inputs for an internal signal. The internal oscillator signal is shaped in the phase-shifting device, 2, the input being from the master oscillator, 5.

The device for separating the extremal values, 4, supplies the higher value of output signals from the RC filters to the indicator.

Thus, there are several oscillators, tuned to the same frequency, and displaced in phase with respect to each other. When one oscillator is generating minimum phase amplitude at the output the other necessarily will be generating its maximum value. Then, using an amplitude analyzer, we always select only the maximum.

Figure 2 shows the mutual disposition of oscillator phases.

The vectors have a rotation rate determined by the frequency of the signal beats. If it is taken that the vector of the separated input signal coincides with

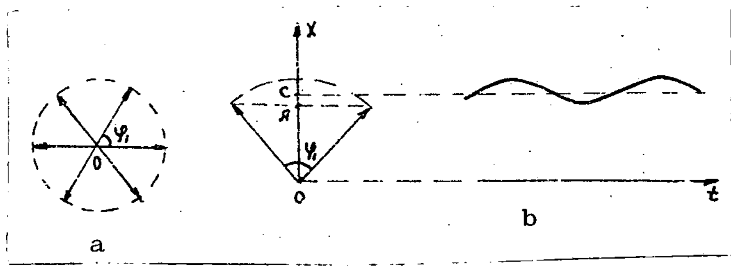

Figure 2.

the OX axis (Figure 2b), we see that the output amplitude for the entire device changes its value significantly.

We have considered the operation of a coherent analyzer, the basis of which /9 is an RC filter. We can, in addition to this method of filtering and separating the difference frequency between input and internal signals, use a resonant circuit consisting of LC components in which the central frequency is not equal to zero as the filtering element. The use of LC filters greatly simplifies tuning the filtering circuit for broader passbands than is the case of RC filters. Figure 3 shows the principle involved in tuning a coherent analyzer with resonant circuits.

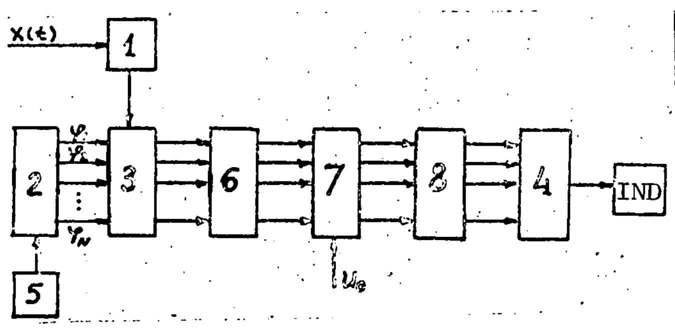

Figure 3.

Units 1 through 5 perform the same functions here as they did in the preceding block diagram. The interaction between the two signals causes resonant filter 6 to generate a signal at the resonant frequency. The phases of the output signals will depend on the phase of the internal signal. A harmonic signal is supplied to the synchronous detector, 7, from the resonant filter. A signal at the same frequency as the resonant frequency from the LC filter is supplied to the other input to this system. The signal generated by the synchronous detector is supplied to the integrator, 8. Signals are supplied to the amplitude analyzer from the integrators.

As we see from the description of the principles involved in tuning the two types of coherent analyzers, the one using the RC filters is much simpler. It /10 is this analyzer that will be given most of the attention in what follows.

We have given a brief description of a coherent analyzer functioning on the sequential principle in which there is one predetermined tuning frequency and a set of phases differing from each other by $\Delta\varphi = 2\pi/N$, where N is the total number of phases of the signal from the internal oscillator.

The internal oscillator frequency can change in the coherent analyzer when there is a predetermined frequency spectrum. The shift in the phase will remain unchanged regardless of the analyzer's tuning frequency.

In this case the equation for the internal oscillator is in the form

$$y(t) = \sum_{m=1,3,5\ldots}^{n} y_1 \left| \begin{array}{l} \sin(m\omega_0 t + 0) \\ \sin(m\omega_0 t + \frac{2\pi}{N}) \\ \sin(m\omega_0 t + 2\frac{2\pi}{N}) \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \sin(m\omega_0 t + 2\pi - \frac{2\pi}{N}) \end{array} \right| \tag{1.4}$$

Let us designate

$$\begin{array}{l} \sin(m\omega_0 t + 0) \\ \sin(m\omega_0 t + \frac{2\pi}{N}). \\ \sin(m\omega_0 t + 2\frac{2\pi}{N}) \\ \ldots\ldots\ldots\ldots\ldots \\ \sin(m\omega_0 t + 2\pi - \frac{2\pi}{N}) \end{array} \left| = \sum_{j=0,\frac{2\pi}{N},2\frac{2\pi}{N}}^{2\pi-\frac{2\pi}{N}} \sin(m\omega_0 t + \varphi_j). \right| \tag{1.5}$$

for all internal oscillator signals in order to shorten the mathematical description.

Accordingly, in the case of a multiphase analyzer we have an equation for the internal oscillator in the following form

$$y(t) = \sum_{m=1,3,5}^{n} \sum_{j=0,\frac{2\pi}{N},2\frac{2\pi}{N}}^{2\pi-\frac{2\pi}{N}} y_m \sin(m\omega_0 t + \varphi_j). \tag{1.6}$$

Disregarding the higher frequencies of the internal oscillator in the first [11] approximation, and setting $y_1 = 1$, we obtain

$$y(t) = \sum_{j=0,\frac{2\pi}{N},2\frac{2\pi}{N}}^{2\pi-\frac{2\pi}{N}} \sin(\omega_0 t + \varphi_j). \tag{1.7}$$

Then the magnitude of the output signal from the analyzer will be found from

$$z(t) = C \sum_{l=0,1,2}^{n} X_l \sin(l\omega t + \varphi_l) \sum_{j=0,\frac{2\pi}{N},2\frac{2\pi}{N}}^{2\pi-\frac{2\pi}{N}} \sin(\omega_0 t + \varphi_j)$$

or

$$z(t) = \frac{c}{2}\sum_{i=0,1,2}^{n}\sum_{j=0,\frac{2\pi}{N},2\frac{2\pi}{N}}^{2\pi-\frac{2\pi}{N}} x_i[\cos(i\omega t+\varphi_i+\omega_0 t+\varphi_j)+\cos(i\omega t+\varphi_i-\omega_0 t-\varphi_j)]. \tag{1.8}$$

The first term under the sum sign cannot be taken into consideration because the component determined by the sum of the frequencies will be filtered out by the next filter.

The result is

$$z(t) = \frac{c}{2}\sum_{i=0,1,2\cdots}^{n}\sum_{j=0,\frac{2\pi}{N},2\frac{2\pi}{N}}^{2\pi-\frac{2\pi}{N}} x_i\cos[(i\omega-\omega_0)t+(\varphi_i-\varphi_j)]. \tag{1.9}$$

We see from Eq. (1.9) that the maximum value of the function will be determined by the conditions $i\omega = \omega_0$

$$z_0 = \frac{c x_i}{2}\sum_{j=0,\frac{2\pi}{N},2\frac{2\pi}{N}}^{2\pi-\frac{2\pi}{N}} \cos(\varphi_i-\varphi_j). \tag{1.10}$$

The maximum for the function z_0 will be reached when the condition is $\varphi_i - \varphi_j = 0$, where $\cos(\varphi_i - \varphi_j) = 1$, depending on the signal phase relationships.

This equality reflects ideal coincidence of the input signal phase with one of the internal oscillator phases. The external signal phase can take any value $\angle 12$ $\varphi_i = 0 + 2\pi$, however.

Since the internal signal phase takes definite discrete values in the range 0 to 2π, that is, $\varphi_N = 2\pi/N$ any input signal phase always will lie alongside one of the internal signal phases. The actual phase between signals will be determined as

$$\varphi = \varphi_i - \left(\varphi_N \pm \frac{2\pi}{2N}\right) \quad \text{or} \quad \cos\varphi = \cos\left(\varphi_i - \varphi_N \mp \frac{2\pi}{2N}\right).$$

The magntidue $2\pi/2N$ characterizes the maximum deviation of the external signal phase.

Now we obtain

$$z_0 = \frac{c x_i}{2}\cos\frac{\pi}{N}. \tag{1.11}$$

Figure 4 shows the frequency and phase characteristic curves for the method of operating a multiphase analyzer we have analyzed.

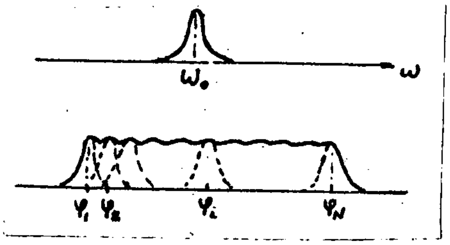

Figure 4.

This method of operating a coherent analyzer can be characterized by the sequential principle of operation with a stationary phase.

A multiphase analyzer with a dynamic phase can be built as well.

Beat frequencies, the result of the interaction of two signals, will appear if there is a signal with frequency ω_0 at the input, and if the internal signal consists of a central frequency ω_0 and additional frequencies positioned at equal, discrete, values $\Delta\omega$ on both sides of the central frequency. Now when there is a gradual temporal change /13 in the phase between the external and internal signals with a positive initial phase in one phase channel there will be a change in the output signal that will be opposite in phase on the other phase channel.

The analytical expression for the analyzer's internal signal will now be in the form

$$y(t) = \sum_{k=0,1,2,3}^{\frac{\Delta\Omega_a}{2\Delta\omega_2}} y_k \sin(\omega_0 \pm k\Delta\omega_2)t. \qquad (1.12)$$

The number of discrete internal signal frequencies will be determined by the analyzer's passband, $\Delta\Omega_a = k\Delta\omega_2$ or $k = \Delta\Omega_a/\Delta\omega_2$. The magnitude $\Delta\Omega_a = \Delta\Omega_{RC}$ is determined by the RC filter passband.

Note that when we design an analyzer passband with a discrete set of frequencies we obtain a resonant characteristic curve for analyzer transmission that is significantly different from the resonant characteristic curve for an RC filter. The analyzer passband designed in this manner will have good rectangularity.

Let us assume that a signal with frequency ω_0 is all that is acting at the input. The phase between the input signal and the analyzer's central frequency is equal to φ.

The interaction between the two signals yields

$$z_o(t) = C \sum_{K=0,1,2,3}^{\frac{\Delta\Omega a}{2\Delta\omega_2}} \frac{X_i Y_K}{2} \cos(\varphi \pm K\Delta\omega_2 t). \qquad (1.13)$$

This expression does not take into consideration signals with frequency $2\omega_0$. They will be filtered out by succeeding filters.

Moreover, the equation that includes internal oscillator signals displaced by a phase angle of $\pi/2$ should be written for this expression. It is comparatively simple to obtain a signal with a phase shift of $\pi/2$. Signals of the same frequency, but with different phases, can be tapped off the branches of the different output triggers of the phase-shifter. It can be taken that the analyzer's output signal is shaped in accordance with two equations /14

$$z_o(t) = \begin{vmatrix} z_o \sum_{K=0,1,2,3}^{\frac{\Delta\Omega a}{2\Delta\omega_2}} \cos(\varphi \pm K\Delta\omega_2 t) & (1.14) \\ z_o \sum_{K=0,1,2,3}^{\frac{\Delta\Omega a}{2\Delta\omega_2}} \sin(\varphi \pm K\Delta\omega_2 t) & (1.15) \end{vmatrix}$$

Thus, if $\varphi = 0$, we obtain the maximum for the function $\cos\varphi = 1$ when $K = 0$, and $\sin\varphi = 0$. The result is failure to consider the second sum and the analyzer's output signal is determined solely by the component $\cos\varphi$.

Since the external signal coincides in phase with the internal signal, which has a central frequency, the analyzer's output signal will be determined only by the value $K = 0$. All other values $K \neq 0$ will yield signals with different beat frequencies. The amplitudinal value of the signals with these beat frequencies cannot exceed the signal value when $K = 0$.

We will find the analyzer in a similar state if $\varphi = \pi/2$. Now $\sin\varphi = 1$ and $\cos\varphi = 0$. Here too there is a state in which the output signal is determined solely by the value $K = 0$.

If φ is somewhere between 0 and $\pi/2$, the signal from the phase channel will take up an intermediate position between the maximum value A_{max} and the value when $\varphi = \pi/4$ when $K = 0$; that is, $\sin\varphi = \cos\varphi = \sqrt{1}/2\, A_{max}$.

At this point the beat frequency for $K \geq 1$ begins to play a significant role. /15 The output signal from the analyzer will be shaped from all signals.

The constant component of the analyzer's output signal will be shaped from the phase channel for K = 0 and from the absolute value of the signal $|\cos k\Delta\omega_2 t|$. The variable component of this signal will have a frequency determined by the summed action of all phase signals.

Figure 5 shows the modular values for the signals from the three phase channels.

Figure 5.

We see from this figure that the analyzer's output signal will be determined primarily by the maximum beat frequency; that is, by that frequency found at the edge of the analyzer's passband.

The frequency of the variable component of the output signal will be equal to double the frequency of the maximum beat frequency. The amplitude value of the signal at this frequency will be the one that will appear most often at the analyzer output. So we can say that K = 3. The internal oscillator signal consists of a central frequency and two frequencies located at the edges of the analyzer's passband.

We have considered the case when the external signal coincides in frequency with the central frequency of the analyzer's internal signal, but in the general case the frequency of the external signal can differ from the central frequency of the internal signal. The frequency of the external signal can lie at the edge of the analyzer's passband, in which case the internal signal with frequency $\omega_0 + k\Delta\omega_2$ will yield a beat frequency with the external signal that will be close to zero. What we obtain here is the fact that the analyzer's output signal /16 will be determined by the central frequency of the internal signal. The beat frequencies that occur between the external signal and the signal from the internal oscillator with its frequency located at the other edge of the passband will be filtered out by the RC filter. They will be beyond the filter passband limits.

The picture is somewhat different if the internal signal is built from frequencies that cover a significantly broader passband than in the case analyzed.

In the earlier case the analyzer passband was determined by the RC filter passband, but here it can be greatly in excess of that passband; that is $\Delta\Omega_a \gg \Delta\Omega_{RC}$ and $\Delta\Omega_a/\Delta\omega_2 = L$, $L > K$. Then the interaction between the two signals will be expressed by the equation

$$z(t) = \sum_{i=0,1,2,3}^{n} X_i \sin(i\omega t + \varphi_i) \sum_{L=0,1,2,3}^{\frac{\Delta\Omega_a}{2\Delta\omega_2}} y_L \sin(\omega_0 \pm L\Delta\omega_2)t. \tag{1.16}$$

The initial phase angle, φ_i, between the signals cannot be taken into consideration in this particular case, but it is not significant. The analyzer's output signal will be determined primarily by those beat frequencies obtained as a result of the interaction between the signals. Moreover, let us hypothecate that the input signal frequency is in the band $\Omega_0 \pm \Delta\Omega_0$.

We obtain

$$z_i(t) = c \sum_{i=0,1,2,3}^{i} \sum_{L=0,1,2,3}^{\frac{\Delta\Omega_a}{2\Delta\omega_2}} \frac{X_i y_L}{2} \left\{ \cos[(\Omega_0-\omega_0)-(\pm\Delta\Omega_c \pm L\Delta\omega_2)]t - \right. \\ \left. - \cos[(\Omega_0+\omega_0)+(\pm\Delta\Omega_c \pm L\Delta\omega_2)]t \right\}. \tag{1.17}$$

We can ignore the second term of the sum if $[\Omega_0+\omega_0] + [\pm\Delta\Omega_c \pm L\Delta\omega_2] > 0$. More precisely, this frequency sum should be larger than the RC filter passband, $\Delta\Omega_{RC}$. /17
We can take it that $\Delta\Omega_{RC} \approx 0$, because $\Delta\Omega_a \gg \Delta\Omega_{RC}$.

$$z_0(t) = c \sum_{i=0,1,2,3}^{n} \sum_{L=0,1,2,3}^{\frac{\Delta\Omega_a}{2\Delta\omega_2}} \frac{X_i y_L}{2} \cos[(\Omega_0-\omega_0)-(\pm\Delta\Omega_c \pm L\Delta\omega_2)]t \tag{1.18}$$

This expression reaches its maximum if the cosine function argument is equal to zero, $[\Omega_0-\omega_0] - [\pm\Delta\Omega_c \pm L\Delta\omega_2] = 0$.

If the central frequencies of the spectra, Ω_0 and ω_0, coincide, that is, if $\Omega_0-\omega_0=0$, then $\Delta\Omega_c = L\Delta\omega_2$. Now the output signal from the analyzer will be shaped from that band of frequencies which is the lesser. If $\Delta\Omega_c > L\Delta\omega_2$, then $A_{out} = f[2L\Delta\omega_2]$, and when $\Delta\Omega_c < L\Delta\omega_2$, $A_{out} = f[2\Delta\Omega_c]$. Ordinarily $\Delta\Omega_c \gg L\Delta\omega_2$.

When the central frequencies do not coincide $\Omega_0 \neq \omega_0$, the analyzer's passband does not change, and is equal to $L \cdot \Delta\omega_2$. Changing the central frequency of the internal signal, the analyzer will investigate the spectrum, $\Delta\Omega_c$, of the passband input signal $L \cdot \Delta\omega_2$.

So we have a broad spectrum of beat frequencies. These signals are fed into RC filters with passband $\Delta\Omega_{RC}$. The analyzer's output signal will be shaped only by those beat frequencies in the band $\Delta\Omega_{RC}$.

Accordingly, the analyzer ouput will have a signal amplitude at a frequency matching the internal signal frequency. Signals of equal magnitude will be present in all analyzer phase channels if the spectral density of the input signal is constant (noise present at the input); that is, $G(\omega)$ = constant. But if the spectral density is not uniform, and if it has an excess over the amplitudes of the other frequencies at some frequency, the signal at the analyzer output will have an amplitude that of this frequency. In other words, if some regular component is present in the noise signal, its amplitude will be present at the analyzer output.

Figure 6 shows the frequency and phase curves for the method analyzed, one with a dynamic phase for the coherent analyzer. This method of coherent analyzer operation can be characterized by the parallel principle of operation. /18

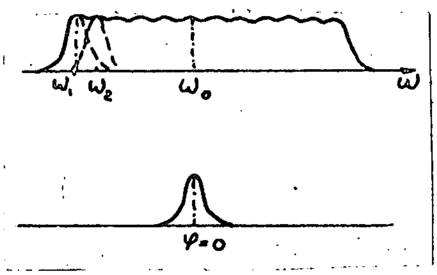

Figure 6.

We have given a brief description of the operation of a multiphase coherent analyzer primarily as applicable to the functioning of a relay type internal oscillator because it is much simpler to obtain a frequency with different phase shifts for this function than for a harmonic signal in a broad range of frequencies. However, the existence in the internal signal of overtones in addition to the fundamental harmonic makes necessary a more detailed consideration of the analyzer operating principle and determination of the main specifications for its components. Moreover, the discrete nature of the operation of virtually all components makes possible a considerable simplification of the specifications for the components, and this provides greater possibilities for building a small, dependable, device.

2. Value of the Phase of the Frequency Components When Investigating the Different Functions of the Physical Processes

The big advantage of the coherent analyzer principle, as compared to all existing analyzers, is the possibility of determining the phase of the frequency component singled out.

The appearance of a maximum at the analyzer output determines the phase of the frequency component automatically. A maximum at the output means that the frequency component of the input signal coincides with the internal oscillator signal in frequency and in phase. If the output signal is formed as a sum with /19 all phase shifts in the internal oscillator signal in order to determine the magnitude of the amplitude of the frequency component, signals must be taken from each phase channel in order to determine the phase of this component.

The accuracy with which the phase of the frequency component is determined will depend on the discreteness of the internal oscillator phase.

Let us consider in somewhat more detail the role of the phase parameters of frequency components when investigating different physical processes.

If a more objective approach is to be made to the investigation of physical processes, more attention must be given to the phase relationships between these components when breaking the analog function down into frequency components. One can invoke many practical examples of our having obtained almost identical spectra during frequency analysis of different functions (Figure 7). In these cases it is necessary to characterize the process in question as a time function, as well as its frequency spectrum. Representation of the process in question in the form of a frequency spectrum enables us to analyze very simply the behavior of the different details of the object and to develop the resonance curves for these objects.

However, occasionally it is difficult to represent the process in question by two curves, because it is necessary to build complex devices to record the process in question and show it graphically in its analog form. A greater quantity of recorded material is required when recording the function in question on different recorders. This may be acceptable in the case of relatively slow and /20 periodic processes that are recorded and processed on the ground, but significant limitations are imposed when the instruments are installed in satellites and spacecraft. On-board instruments for data recording should take up a minimum of space.

Expansion of the spectrum of recorded processes increases the speed of the recorders, and this, in turn increases the quantity of recorded data to the point where it is difficult to process. This relates to processes approaching random ones.

We see a completely different picture when the process in question is represented by using frequency and phase spectra. Here too the results of the research can be represented by two curves, but we can use the frequency and phase spectra as the basis for reproducing the function in question with the help of elementary

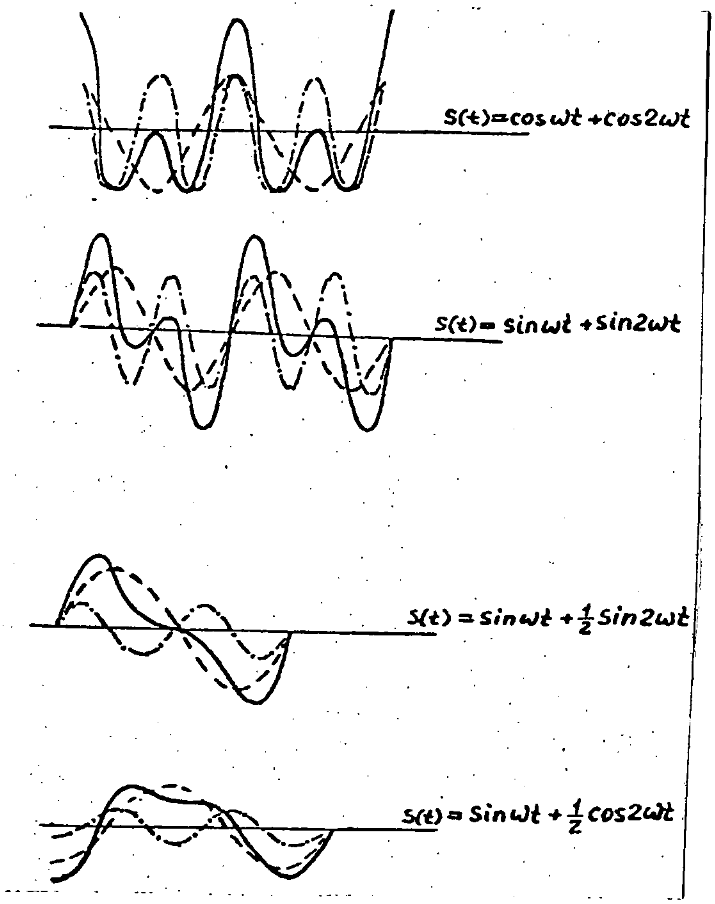

Figure 7.

constructions.

We obtain a big advantage in the recording of these spectra when we represent the function in question in the form of frequency and phase spectra. We can use the same instruments and devices to record the phase spectrum and the frequency spectrum. Using $G(\omega)$ and $\varphi(\omega)$ expansions we can obtain the primary time function $x(t)$. But this assertion will be valid only if the passband for the coherent analyzer will tend to zero, and, in this connection, the number of frequencies analyzed, to infinity; that is

$$x(t) = \left\{ \begin{matrix} G(\omega) \\ \varphi(\omega) \end{matrix} \right\} \longrightarrow \gamma(t)$$

$$\lim_{\substack{\Delta f \to 0 \\ n \to \infty}} \gamma(t) = x(t)$$

Moreover, the advantages derived in investigating different signals when recording the phase of the frequency component can be seen from the example of the discovery of a new photographic method, holography. Conventional photography records the amplitude distribution of the intensity of light waves reflected from the surface of an object, but holography records the reflected intensity of the light, and its phase [9, 10]. /22

We have considered the case when two functions $A = f(\omega)$ and $\varphi = f(\omega)$, are used simultaneously to investigate signals. But the phase function, which depend on frequency, or on time, can be used successfully to investigate unknown signals.

It is possible to obtain four phase dependency methods:

1. $\varphi = f(\omega)$ when $A = \text{var}$, $\omega = \text{var}$, (2.1)
2. $\varphi = f(\omega)$ when $A = \text{const}$, $\omega = \text{var}$, (2.2)
3. $\varphi = f(\omega)$ when $A = \text{var}$, $\omega = \text{const}$, (2.3)
4. $\varphi = f(\omega)$ when $A = \text{const}$, $\omega = \text{const}$. (2.4)

The first method characterizes the analyzer operating mode to obtain the phase spectrum function. It can be used successfully in combination with the function $A = f(\omega)$. The relationship between these functions was considered above.

The phase spectrum function obtained by the second method will contain much more information if amplitude changes in the output signal are converted into a phase function. This conversion can be done by negative feedbacks, or by associating these changes with the function $\varphi = f(\omega)$ through a predetermined function and thus obtain a new phase function $\varphi_n = f(\omega, A)$.

Let us consider two examples in order to determine the possibilities of making the conversion.

Known is the fact that a continuous function $x(t)$ can be replaced by a set of its discrete values, determined at equal time intervals, Δt, Figure 8a. /23

The spectral function of the discrete value of a unit amplitude is in the form

$$q_1(\omega) = \frac{\sin \frac{\omega\tau}{2}}{\frac{\omega\tau}{2}} e^{-i\frac{\pi}{2}}. \quad (2.5)$$

The function $x(t)$ then can be represented by a set of discrete values (the

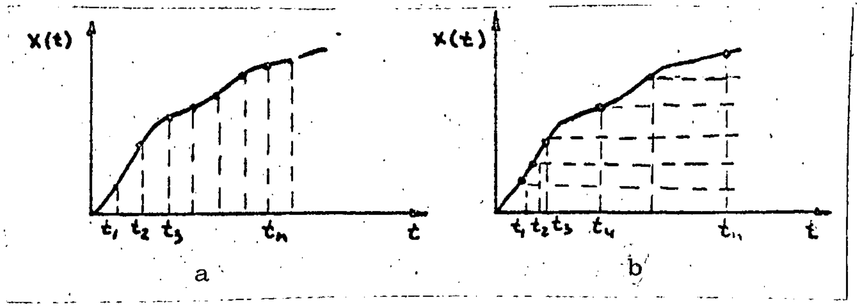

Figure 8.

Kotel'nikov theorem)

$$x(t) = \sum_{n=0,1,2}^{\infty} x_n(t_n) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \sum_{n=0,1,2}^{\infty} X_n(t_n)\, e^{-i\omega n\Delta t}\, e^{-i\frac{\pi}{2}}\, e^{i\omega t}\, d\omega ,$$

$$x(t) = \frac{e^{-i\frac{\pi}{2}}}{2\pi}\int_{-\infty}^{+\infty} \sum_{n=0,1,2}^{\infty} x_n(t_n)\, e^{i\omega(t-n\Delta t)}\, d\omega . \qquad (2.6)$$

We see that all the data on the current value of the function x(t) is contained in the change in the amplitudes of the frequency components of the current spectrum. The magnitude $n\cdot\Delta t$, which is constant, determines the phase of these components. (Actually, the phase of the frequency components is not constant if a strict approach is taken to this expression. It will change discretely from one measurement, t_n, to another, t_{n+1}. We shall disregard this for now. We are interested in the amplitude of the frequency components).

Then we can write /24

$$x(t) \longrightarrow \frac{1}{2\pi}\int_{-\infty}^{+\infty} g_2(\omega, x_n)\, e^{i\omega t - \varphi_n}\, d\omega \qquad (2.7)$$

There is another type of conversion that can be suggested in addition to this type of conversion of an analog signal that more neatly approaches the adaptive method of converting data, and is shown in Figure 8b.

Let us say we have the same function, x(t). This function can be replaced by unit discrete changes at predetermined times

$$x(t) = 1(t_1) + 1(t_2) + 1(t_3) \;.........\; 1(t_n). \qquad (2.8)$$

The spectral function then will be determined by the sum of the spectral functions of the individual unit changes

$$S_1(\omega) = \int_0^{\infty} 1(t_n)\, e^{-i\omega t_n}\, e^{i\omega t}\, dt = e^{-i\omega t_n}\int_0^{\infty} 1(t_n)\, e^{i\omega t}\, dt ,$$

but

$$\int_0^\infty 1(t_n) e^{i\omega t} dt = \frac{1}{i\omega}.$$

Consequently

$$S_1(\omega) = \frac{1}{i\omega} e^{-i\omega t_n}.$$

The spectral function of the signal, x(t), is equal to

$$S_2(\omega) = \frac{1}{i\omega} e^{-i\omega t_1} + \frac{1}{i\omega} e^{-i\omega t_2} + \frac{1}{i\omega} e^{-i\omega t_3} + \ldots\ldots$$

$$S_2(\omega) = \frac{1}{i\omega} \sum_{n=1,2,3}^{\infty} e^{-i\omega t_n}.$$

Then

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{1}{i\omega} \left[\sum_{n=1,2,3}^{\infty} e^{-i\omega t_n} \right] e^{-i\omega t} \, d\omega$$

or

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S_1(\omega) \sum_{n=1,2,3\ldots}^{\infty} e^{-i\omega t_n} \, d\omega. \qquad (2.9)$$

We see from this expression that the current value of the signal, x(t), is contained in the phase function $i\omega t_n$.

Thus, we conclude that the analog function, x(t), can be replaced by the amplitude-frequency spectrum, as well as by the phase-frequency spectrum. The amplitude of the frequency components remains unchanged. /25

This conversion can be written as follows in general form

$$G(\omega) e^{i\varphi(\omega)} = A(\omega) e^{i\xi(\omega)}. \qquad (2.10)$$

Here the parameter carrying the data on the behavior of the function is in $G(\omega)$ and $\xi(\omega)$.

This expression can be used to convert the spectra. When $A(\omega)$ = constant

$$e^{i\xi(\omega)} = \frac{G(\omega)}{A(\omega)} e^{i\varphi(\omega)} \quad \text{or} \quad i\xi(\omega) = i\varphi(\omega) + \ln \frac{G(\omega)}{A(\omega)}.$$

The modulus is

$$|\xi(\omega)| = \sqrt{\varphi^2(\omega) + \ln^2 \frac{G(\omega)}{A(\omega)}} \,. \tag{2.11}$$

Let us approach the question of converting the spectra from another direction.

Conventionally, if the function in question is even it must be distributed by sine component, and if odd by cosine component. The intermediate function can be distributed by either. Here there is a phase angle that is constant for all frequencies. Its sole dependence will be on the amplitude of the frequency components. The connection between amplitudes and phase angle can be determined as

$$\varphi = \operatorname{arctg} \frac{a_k}{b_k}, \quad \text{where } a_k = \frac{1}{T}\int_0^T x(t) \sin \omega t \, dt, \quad b_k = \frac{1}{T}\int_0^T x(t) \cos \omega t \, dt.$$

If the question of the behavior of the function x(t) in time $t < 0$ (relative to the origin of the reading) is of no interest to us, we have the right to select any function in this time interval and convert the function x(t) when $t > 0$ into even or odd. We merely determine the value of the phase not dependent on the frequency by this operation, but we can, with the equal success, consider the function x(t) as neither even nor odd, and assign any phase shift. If a phase shift that does not depend on the frequency is selected, we will have an amplitude frequency spectrum for the signal in question no different from already known developments. /26

We will find the picture to be completely different if the phase shift is a function of the frequency. Observed here is a change in the amplitude spectrum of the frequencies.

We can write the spectrum of the function x(t) as

$$G_{ph}(\omega) = \frac{1}{T}\int_0^T x(t)\, e^{i\omega t + i\alpha(\omega)}\, dt. \tag{2.12}$$

Let G_{ph} = constant. We must have

$$G_{ph}(\omega) = const = e^{i\alpha(\omega)}\,\frac{1}{T}\int_0^T x(t)\, e^{i\omega t}\, dt, \tag{2.13}$$

for this to be so, but $\frac{1}{T}\int_0^T x(t)\, e^{i\omega t}\, dt = G_A(\omega)$ is the amplitude spectrum of the function x(t). In this case

$$G_{ph}(\omega) = G_A(\omega)\, e^{i\alpha(\omega)},$$

$$\alpha(\omega) = -i\,\ln\frac{G_{ph}(\omega)}{G_A(\omega)}. \tag{2.14}$$

Thus, we find that all data on the behavior of the function x(t) is contained in the phase of the frequency components.

But these examples have provided nothing to prove Eq. (2.10). The solution must be approached with greater mathematical discipline. We have merely shown that this conversion is possible. /27

The third and fourth methods for obtaining phase relationships enable us to establish the nature of the signal in question.

All of today's research on input signals is devoted to breaking them down into frequency components. But any frequency component separated by an analyzer can be the product of carrier frequency modulation, and modulation can be of different types, so the analyzer, while separating the frequency components, failes to provide information as to the nature of these components. Neither the fundamental frequency of the signal in question, nor the type of modulation of this component, are known.

Operation of the analyzer in the fourth mode enables us to separate a frequency (phase) modulated signal. But the frequency constancy of this mode fails to tell us if the analyzer is functioning on a single fixed frequency. The analyzer has a broad passband in this mode and can scan its central frequency in the band in question. Fourth mode operating conditions are satisfied when the fundamental frequency components responding to the FM (PM) oscillation are within the analyzer's passband, and we have the modulation function of this oscillation at the output. Thus, we can replace a whole set of frequency components with one modulation function and we can easily present them in mathematical form for future analysis.

The third analyzer operating mode can be used to represent the frequency components of an amplitude-modulated oscillation.

Here a modulation function can be obtained for $A = f(t)$ when $\varphi = f(t) =$ constant, $\omega =$ constant.

There are frequency components formed by AM and FM (PM) oscillations in the analyzer passband if it is found that $A = f(t)$ and $\varphi = f(t)$.

This ambiguity can be eliminated by using the analyzer operating mode in which signals can be converted from one type of modulation to another. An AM signal can be converted into an FM signal, for example. /28

This analyzer mode permits us to select the most convenient mathematical form for the representation of the spectrum in question. Practice can show that the description of the mathematical function of the signal in question with AM will be very complex, and that subsequent study of this modulation function will be very difficult. In such cases it is desirable to represent the signal in question by the result of frequency (phase) modulation.

Let us consider the conditions for the conversion of AM into FM (PM) in order to bring about this analyzer operating condition. The amplitude changes in the input signal must be converted into changes in the phase (frequency) of the internal oscillator.

In the case of an AM signal

$$(1 + m \cos \Omega t) \sin \omega_1 t, \tag{2.15}$$

and for an FM (PM) signal

$$\sin [\omega_2 t + \varphi(t)]. \tag{2.16}$$

Then

$$(1 + m \cos \Omega t) \sin \omega_1 t = \sin [\omega_2 t + \varphi(t)], \tag{2.17}$$

and

$$\arcsin [\sin \omega_1 t + m \cos \Omega t \sin \omega_1 t] = \omega_2 t + \varphi(t)$$

or

$$\varphi(t) = (\omega_1 - \omega_2)t + \arcsin [m \cos \Omega t \sin \omega_1 t]. \tag{2.18}$$

If the carrier (central) frequencies of both modulations coincide, that is, if $\omega_1 = \omega_2$

$$\varphi(t) = \arcsin [m \cos \Omega t \sin \omega_1 t]. \tag{2.19}$$

Moreover, when the analyzer filter's central frequency is $\omega_2 = 0$, we obtain

$$\varphi(t) = \arcsin (m \cos \Omega t). \tag{2.20}$$

but

$$\arcsin x = x + \frac{x^3}{2 \cdot 3} + \frac{1 \cdot 3 \cdot x^5}{2 \cdot 4 \cdot 5} + \frac{1 \cdot 3 \cdot 5 \cdot x^7}{2 \cdot 4 \cdot 6 \cdot 7} + \ldots\ldots$$

$$\varphi(t) = m \cos \Omega t + \frac{m^3 \cos^3 \Omega t}{2 \cdot 3} + \frac{1 \cdot 3 \cdot m^5 \cos^5 \Omega t}{2 \cdot 4 \cdot 5} + \ldots\ldots \tag{2.21}$$

When $m \ll 1$ /29

$$\varphi(t) \approx m \cos \Omega t. \tag{2.22}$$

We have confirmed that the spectrum of an amplitude-modulated oscillation in the case of small values is similar to the spectrum of a frequency (phase)-modulated oscillation.

So it is very simple to replace the complex modulation function in the AM case by the simpler FM (PM) one because the frequency-modulated oscillation has more harmonics.

Moreover, attention should be drawn to the fact that when the function $\varphi = f(t)$ is recorded the analyzer can be used successfully as a selector to separate the useful harmonic signal from the noise in a narrow passband. This cannot be done with optimum filtering and the correlation method because the amplitude curves for narrow-band noise are similar to those for a harmonic signal. The significant difference in these signals is in the phase distribution. Phase, in the case of narrow-band noise, is uniformly distributed between 0 and 2π.

Figure 9 shows known curves for the density of the distribution of the phase of a signal plus noise for different values of the signal/noise ratio, s. We see from the graphic that values $s \ll 1$ can provide reliable evidence of the existence of a harmonic signal at the input [10, 13].

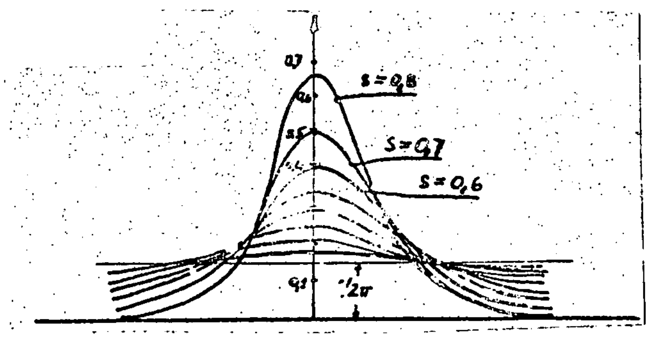

Figure 9.

We have considered, very brief- /30
ly, the possibilities available to the researcher when he uses the phase characteristics of unknown physical processes.

The arsenal of measuring devices today has no devices capable of constructing phase spectra in a broad band of frequencies. The multiphase coherent analyzer will make it possible to solve the problems posed.

3. Two Methods of Building a Multiphase Coherent Analyzer

In section 1, in our consideration of coherent analyzer operating principles, we devoted our attention solely to the term containing the $\cos\varphi$ factor, on which the result of separation of the frequency components is heavily dependent. But if coherent analyzer operating principles are to be presented in more detail we must include as well the constant component in the internal oscillator signal spectrum, y_0.

We took it that there was no constant component in the input signal. It was assumed that there are different signals with frequency as low as desired. We have limitations in terms of analysis time in the case of such an assumption.

The frequency, $i\omega$, can take values as close to zero as desired in the expression for the external signal, $x(t)$. Upon multiplying the two functions we obtain the summand

$$z(t) = c y_0 \sum x_i \sin(i\omega t + \varphi_i). \qquad (3.1)$$

This summand has frequency $\omega_i \approx 0$. But signals that fix the coincidence of the internal and external signal frequencies contain a like frequency value. There is an ambiguity in the determination of the frequency components of the input signal. The energy of frequencies $\omega_i \approx 0$ will be present in the analyzer out- /31
put signal.

The RC filter passband must be significantly below the minimum frequency of the input signal spectrum in order to in some way reduce the error in the ouput signal.

Thus, when there is a constant component in the function of the internal oscillator we have a connection between the input signal spectrum and the analyzer passband. In this case maximum filter passband, $\Delta\Omega_{RC} \leq \omega_i$, is necessary, and n switchings are needed to survey the frequency spectrum, $\Delta\Omega_{in}$

$$n \geq \frac{\Delta\Omega_{in}}{\Delta\Omega_{RC}} = \frac{\omega_{max} - \omega_{min}}{\omega_{min}}. \tag{3.2}$$

This shortcoming appears in particular in the analysis of the spectrum of low and extremely low frequencies.

The constant component in the internal function must be eliminated as shown in Figure 10 in order to investigate the low-frequency components of the sepctrum.

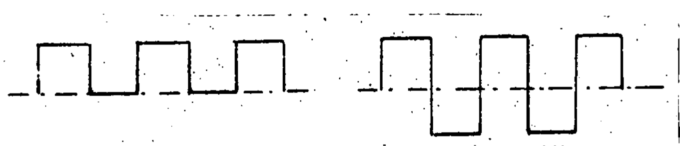

Figure 10.

An internal signal without a constant component can be used in a broader band of frequencies. It can be used successfully to separate relatively high frequency components of the input signal in addition to the low-frequency part of the spectrum. The relative complexity of doing so with electronic components is the drawback in the second method. Whereas only a switching unit is needed for the first method, the second requires two similar units and they should be matched, not only with each other, but with the analyzer input device as well. /32

4. <u>Selection of Number of Phases</u>

In our brief description of the principle of operation of the analyzer as a whole we said that the output amplitude was heavily dependent on the phase difference between the two oscillations.

We shall, in what follows, analyze and select the parameters relative to the fundamental harmonic of the internal oscillator, and we shall take the amplitude value of this harmonic as unity. This will simplify matters. Then the magnitude of the output signal will be determined by the formula

$$z_0 = \frac{cx_i}{2} \cos \varphi.$$

The value $z_0 = cx_i/2$ will reach a maximum only when the phase difference between the two oscillations is zero. And because this difference can take all values from 0 to 2π with equal probability, then so too can the value $z_0 \pm cx_i/2$. We should break the whole phase range of the internal oscillator frequency down into discrete intervals in order to rid ourselves of a random phase value. Now the magnitude z_0 will depend on $\Delta\varphi = 2\pi/N$, where N is the number of internal oscillator signal phases. The random phase of the input harmonic can, in this case, be in phase with the internal signal with a high degree of probability. The range of change in phase between the two signals will be $\Delta\varphi$, and this means that the change in the magnitude z_0 will be much less

$$z_0 \sim \frac{cx_i}{2} \div \left|\frac{cx_i}{2}\right. \cos \Delta\varphi . \tag{4.1}$$

All these arguments relate primarily to the case of absolutely stable signal generators. The phase between the signals is fixed when the instrument is energized and remains stable throughout the measurement. The value of $\Delta\varphi$ must be made as small as possible in order to obtain the amplitude of the separated frequency component with more reliability. /33

But it is a practical impossibility to keep two signals absolutely stable. There will be a beat process taking place between the two signals, and the phase between the two oscillations will change with the beat frequency for these oscillations

$$\cos \varphi = \cos \Delta\omega t. \tag{4.2}$$

Let us consider the behavior of the amplitude of the output signal when there is a frequency difference, $\Delta\omega$, between the two oscillations and the phase discreteness, $\Delta\varphi$.

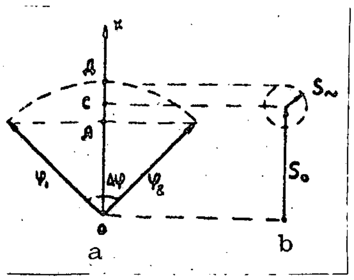

Figure 11.

Figure 11a shows the situation for two signals with difference phase shifts.

Each phase vector, φ_1, φ_2... and so forth, rotates with frequency $\Delta\omega$ when there is a frequency difference between the two signals, $\Delta\omega$. When there are N phases, the magnitude of the output signal, determined by the projection on the OX axis, will have a constant component, S_0, and a variable component, $S_\sim$, Figure 11b.

As we see from the figure, the variable component of the output signal will have a frequency that will depend on the frequency of the passage of the OX axis by the phase vectors. The frequency of the variable component will equal $N\Delta\omega$. The amplitude value of the constant and variable components can be determined from elementary trigonometric relationships for the triangle formed by the phase vectors and the OX axis with angle $\Delta\varphi/2$.

The magnitude of the constant component therefore is determined by the OC vector, that of the variable component by the AC vector. We now can write the equation for the output signal by proceeding from the fact that OC = AO + AC + AD/2. /34

$$AO = \frac{cX_i}{2}\cos\frac{\Delta\varphi}{2}, \quad AD = \frac{cX_i}{4} - \frac{cX_i}{4}\cos\frac{\Delta\varphi}{2}, \quad OC = \frac{cX_i}{4}\left(1+\cos\frac{\Delta\varphi}{2}\right) = \frac{cX_i}{2}\cos^2\frac{\Delta\varphi}{2},$$

$$z_0(t) = \frac{cX_i}{2}\cos^2\frac{\Delta\varphi}{2} + \frac{cX_i}{2}\sin^2\frac{\Delta\varphi}{2}\cos N\Delta\omega t. \tag{4.3}$$

The presence of a variable component in the output signal will be the result, primarily, of the signal frequency difference, and it is a practical impossibility to do away with this component completely because of the impossibility of arriving at an absolute match in the frequencies of two different oscillators. This match is uncontrolled and can occur only during individual time segments.

The RC filter will have selective properties for the variable component of this signal. The corresponding formula for finding the transfer coefficient for the variable component, $S_\sim$, is

$$K_{t\sim} = \frac{1}{\sqrt{1+\tau^2(N\cdot\Delta\omega)^2}}. \tag{4.4}$$

If the two transfer coefficient formulas for separating the fundamental frequency, K_{tf}, and $K_{t\sim}$ are compared, we see that $K_{t\sim}$ can be determined by the frequency $N\Delta_{\omega}$, and that K_{tf} will be determined by the frequency $\Delta\omega$.

So we can write the general equation

$$z(t) = \frac{c x_i}{2}\left[\cos^2\frac{\Delta\varphi}{4} + \sin^2\frac{\Delta\varphi}{4}\cos N\Delta\omega t\right]\cos\Delta\omega t. \qquad (4.5)$$

This relationship is shown in Figure 12.

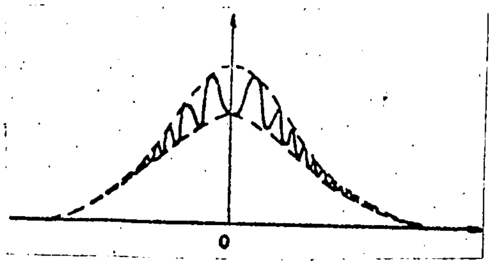

Figure 12.

The variable component $S_{\sim}$ will introduce the greatest errors at the time of fine tuning of the internal oscillator to the frequency component of the input function. We can obtain as small a /35
$\Delta\omega$ value as desired at that time, one that will vanish with absolute tuning. The indicator in this case will show a slow oscillation of the output signal, determined by the variable component, within certain limits.

The corresponding expressions are

$$\Delta\Omega_{RCO} = \Delta\omega \quad \text{and } \Delta\Omega_{RCN} = \frac{\Delta\omega}{N}. \qquad (4.6)$$

when the passband is fixed at the 0.7 level for S_O and S_N.

Thus, the existence of two resonances when tuning to a predetermined frequency can provide significant advantages when analyzing signals. The maximum value of the constant component can be used to determine the fact that the frequency component of the input signal is in the passband $\Delta\Omega_{RCO}$. The appearance of slow deviations in output amplitude is a second criterion showing that the frequency component of the input signal is fixed in the $\Delta\Omega_{RCN}$ band.

So we have arrived at the fact that the constant component of the ouput signal can be found from the expressions at (4.3)

$$S_O = \frac{cx_i}{2}\cos^2\frac{\pi}{2N}, \qquad (4.7)$$

and the variable component has the form /36

$$S_{\sim} = \frac{cx_i}{2}\sin^2\frac{\pi}{2N}, \qquad (4.8)$$

The presence in the analyzer's output signal of a variable component causes an error in finding the magnitude of the constant component signal.

The magnitude of the error in the output signal will be determined by the relationship

$$\eta = \frac{S_{\sim}}{S_0} 100\% = \frac{\sin^2 \frac{\pi}{2N}}{\cos^2 \frac{\pi}{2N}} 100\% = 100\% \, tg^2 \frac{\pi}{2N} \qquad (4.9)$$

The curve of this function is shown in Figure 13.

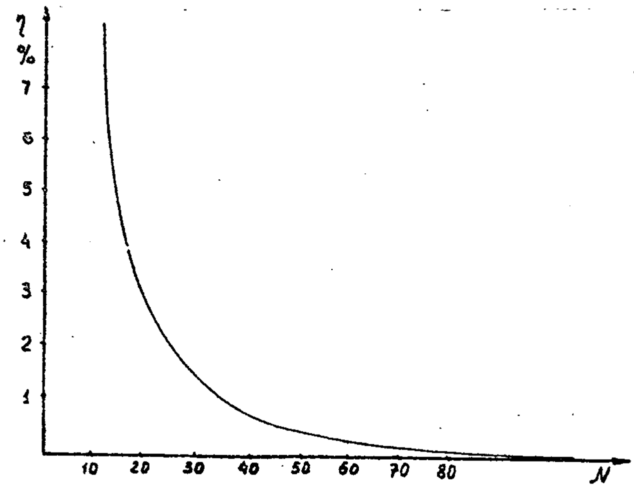

Figure 13.

We see that we must take a value $N > 30$, where the error will be less than 1 percent, if we are to increase the accuracy in determining the output signal, and this is entirely feasible in different devices.

5. Accuracy in Determination of the Phase of the Frequency Component /37

The possibility of analyzing unknown physical processes using frequency and phase spectra results in obtaining much more complete information on the object in question.

Let us see just how accurately we can determine the phase of the frequency component of the input signal. Phase determination accuracy will depend on just how much one phase channel differs from another in its output signal.

We already have established that when the frequencies of the internal and external signals are matched the magnitude of the analyzer's output signal will depend on the phase angle between them. In this case, one phase channel will differ from the other in its output signal by the magnitude of the phase discreteness

$$\Delta z = z_i(t) - z_{i+1}(t). \qquad (5.1)$$

But $z_i(t)$ will correspond to the magnitude $\varphi = 0$, that is, $z_i \doteq z_0$. This can be represented in relative magnitudes in the form

$$\frac{\Delta z}{z_0} = 1 - \cos \Delta\varphi \qquad \text{or} \qquad \frac{\Delta z}{z_0} = 1 - \cos \frac{2\pi}{N}. \tag{5.2}$$

In the case of large values, $N > 10$, we can write

$$\cos \frac{2\pi}{N} \approx 1 - \left[\frac{2\pi}{N}\right]^2 \cdot \frac{1}{2} \tag{5.3}$$

We must have a magnitude Δz as large as possible in order to distinguish one phase channel from the other. This magnitude determines phase accuracy.

We conclude that the magnitude is

$$\frac{\Delta z}{z_0} \approx \frac{2\pi^2}{N^2}. \tag{5.4}$$

The formula obtained establishes the inverse proportionality between the relative error and N. This is in complete contradiction with accuracy in determining the amplitude of the frequency component. /38

Limitations must be imposed on the magnitude Δz if the two contradictory requirements are to be satisfied. The sensitivity of the phase indicator circuit must be increased considerably because the magnitude Δz decreases with increase in N.

As a practical matter, the solution to this problem can be approached from both directions. We can make the circuit for recording Δz sensitive, and by so doing we can determine N, or we can calculate the accuracy in the determination of the amplitude of the frequency component in terms of the N magnitude selected.

We already have found that accuracy in determination of the input signal phase will depend on the discreteness of the internal signal phase. But building an analyzer with number of phases greater than 100 is in practice, a complicated business. The fact that the phase discriminator, the role of which is played by the amplitude analyzer, functions as a linear device, and not as a threshold circuit, can be used to increase accuracy in determining the input signal phase. The signal in this case will appear in several phase channels, rather than in one, at the output of the phase discriminator.

If φ_{in} is between φ_i and φ_{i+1}, the output signals from the phase discriminator will equal $A_i = A_{i+1}$. We can say with confidence that the phase of the input signal is equal to $\varphi_{in} = \varphi_i + \varphi_{i+1}/2$ when these signals are equal. But if $A_i \neq A_{i+1}$, the value φ_{in} will approach φ_i or φ_{i+1}, when $A_i > A_{i+1}$ $\varphi_{in} \to \varphi_i$ and when $A_i < A_{i+1}$ $\varphi_{in} \to \varphi_{i+1}$. In this case precise determination of phase will depend entirely on the A_i to A_{i+1} signal ratio.

We have a signal $A_i = A_0$ at the phase discriminator output when φ_{in} coincides exactly with φ_i. And A_0 always is equal to $A_0 = A_i + A_{i+1}$, whatever the value of φ_{in}.

If it is accepted that the change in $\Delta\varphi_i$ takes place with respect to φ_i, we can write $\varphi_i + \Delta\varphi_i = \varphi_{i+1}$. The value of $\Delta\varphi_i$ can be expressed in terms of the amplitude when the phase is φ_{i+1}, $\Delta\varphi_i = (A_{i+1}/A_0)\Delta\varphi$, where $\Delta\varphi = \varphi_{i+1} - \varphi_i$. When $A_{i+1} = 0$, we obtain $\varphi_{in} = \varphi_i$. /39

We can, in principle, find $\Delta\varphi_{i+1}$ with respect to φ_{i+1}. Then $\varphi_{i+1} - \Delta\varphi_{i+1} = \varphi_i$ and $\Delta\varphi_{i+1} = (A_i/A_0)\Delta\varphi$.

We must know A_i, A_{i+1}, and A_0 in this case in order to establish the phase value $\Delta\varphi_i(\Delta\varphi_{i+1})$. A simple electronic device can be used to find the interconnection between these signals.

6. Influence of Filter Passband Width on Accuracy in Determining the Phase of the Frequency Component

We had assumed, in our determination of the phase of the frequency component, that the coherent analyzer filter passband was relatively narrow. Only one spectral component was passed. In fact, coherent analyzer filters, no matter how narrow we make them, have a definite passband passing all frequency components lying on either side of the spectral component in question because the signals in question usually have continuous spectra.

Figure 14 shows the assumed spectral curve $A(\omega)$ for the signal in question and the location of the filter passband.

We see that in the case of resonant frequency, ω_0, tuning of the coherent analyzer signal with frequencies ω_1 and ω_2, and amplitudes A_1 and A_2, respectively, will pass in addition to the fundamental frequency. /40

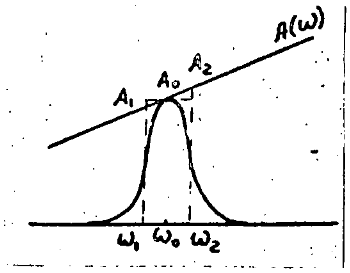

Figure 14.

Let us represent the frequency curve for the filters as a rectangle. Upon conversion of the signal frequencies, ω_1, ω_0, and ω_2, the signal with frequency ω_0 will be converted into a constant component, the others into variable components, and their frequencies will be $\Delta\omega_1 = \omega_0 - \omega_1$ and $\Delta\omega_2 = \omega_2 - \omega_0$, $\Delta\omega_1 = \Delta\dot{\omega}_2$.

The difference between the signals is because of the opposite phases involved. The amplitude of the variable component will be $A_2 - A_1 = \Delta A_a$. We have $\Delta A_a = 0$ when $A_1 = A_2$.

Note that components A_1 and A_2 can differ in phase, as well as in amplitude. Then the magnitude ΔA_a will have a much more complex dependency on the phases of signals A_1 and A_2. We can ignore the magnitude ΔA_a if we take it that the filter passband is narrow enough to cause a considerable phase difference in the frequency components within the band limits.

This question requires special consideration, so we shall not pause to consider it here. We shall assume that the frequency components A_1 and A_2 have identical phases.

The variable component ΔA_a will introduce an error into the consideration of the phase of signal A_0, so the readings of the coherent analyzer can change. This assertion is valid only if the error exceeds the threshold level, determined by the discreteness of the analyzer's phase.

We know that one of the internal analyzer signals is in phase with the input signal during the isolation of amplitude A_0. Signals with amplitudes less by the magnitude $\Delta A_{ph} = A_0 - A_0 \cos \Delta\varphi$, where $\Delta\varphi$ is the discreteness of /41 the analyzer's phase, will be present at the analyzer outputs on either side of the main signal.

So if $\Delta A_a > \Delta A_{ph}$, we will observe a periodic change in phase at the analyzer output. But if $\Delta A_a < \Delta A_{ph}$, the influence of the components A_1 and A_2 cannot be included. The boundary condition is the equality $\Delta A_a = \Delta A_{ph}$, that is, $\Delta A_a = A_0 - A_0 \cos \Delta\varphi$, or when $\Delta\varphi \approx 0$

$$\Delta A_0 \approx A_0 \Delta\varphi^2. \qquad (6.1)$$

Eq. (6.1) shows that the influence of adjacent harmonics is reduced when the isolated frequency component amplitude values are large. The influence of the components A_1 and A_2 is increased when A_0 values are small. Consequently, by increasing the gain factor for the input signal preamplifier we can avoid the influence of the components A_1 and A_2.

This condition, moreover, plays a positive role in signal analysis because in this case attention must be given to the frequency components with relatively large amplitude. The small components usually are ignored during analysis.

7. Dischargable filter

Let us consider the behavior of a filter when there is a "step" law change in the internal oscillator frequency. Each step is a predetermined internal oscillator frequency, with the result that we have a discrete frequency change.

We have assumed, in our consideration of coherent analyzer operating principles, that the internal oscillator frequency comprises the fundamental, the first harmonic, of the signal, as well as the third, fifth, and so on. But this is valid only providing that the time to integrate the product of the two functions is long enough. More precisely, the internal oscillator spectrum takes shape in the form of individual frequency components during the analysis. /42

We know that the spectrum of a limited pulse sequence can be determined by the total number of pulses flowing in a predetermined time interval.

There will be a resonance curve with band $\Delta\omega_{RC}$ at the output if the internal oscillator signal consists of individual frequency components. This band is fixed in the main RC filter. Broadening of the passband of the resonance curve at the corresponding band takes place when there is a certain spectrum of frequencies, $\Delta\Omega_h$, in the internal oscillator signal. If $\Delta\Omega_h \gg \Delta\omega_{RC}$, the filter bandpass can be considered to be $\Delta\Omega_h$. When $\Delta\Omega_h + \Delta\omega_{RC}$, the filter passband will be determined by the sum $\Delta\Omega_h + \Delta\omega_{RC}$.

So, if after the passage of some period of time, Δt_0, during which the filter's output amplitude is formed, we discharge the energy that has accumulated in the capacitance, and if, in the next interval, Δt_0, isolate the next frequency, we will, by taking the value of the output signal before discharging the filter, have fine tuned the analyzer to the specified frequency. The resonance frequency of filter tuning will not shift during time Δt_0, as is the case in the dynamic mode. Analyzer passband is expanded if the time to analyze one frequency component is reduced.

Let us use the method already developed in radar theory to solve the problem of changing the coherent analyzer passband in the case of a variable RC filter discharge time.

We know that the spectrum of a sequence of square pulses can be described by the equation

$$G_p(\omega) = \frac{S_0(\omega)}{N_p \tau_p} \cdot \frac{\sin N_p \frac{T_p}{2} \omega}{\sin \omega \frac{T_p}{2}} \qquad (7.1)$$

where /43

$S_0(\omega)$ is the spectrum for a single pulse;

N_p is the number of pulses;

τ_p is pulse length;

T_p is the pulse repetition rate.

As we see from the formula, energy will begin to be concentrated near the frequency $\omega = k_\tau \omega_p = k_\tau \ (2\pi/T_p)$ with increase in the number of pulses, N_p. Figure 15 shows the amplitude spectrum for $N_p = 3$ (a) and $N_p = 9$ (b).

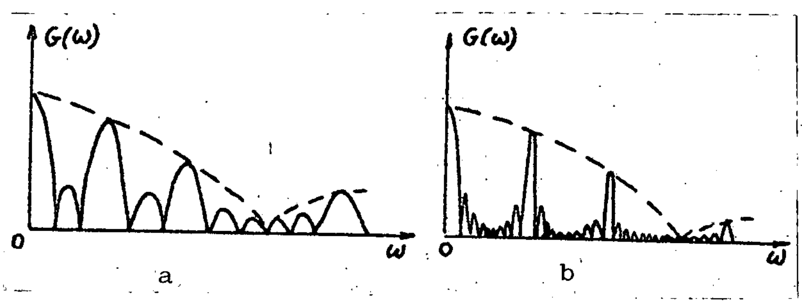

Figure 15.

The middle of the main lobes corresponds to frequency

$$\frac{\omega T_p}{2} = k_\tau \pi, \qquad k_\tau = 0, 1, 2, 3... \qquad (7.2)$$

The passband at the zero level is equal to $\Delta\omega_0 = 2\omega N/N_p$, and the maximum amplitude at the 0.7 level is

$$\Delta \omega_{0.7} = \frac{0.9\omega N}{N_p} = \frac{\omega N}{N_p}. \tag{7.3}$$

The important fact is that the width of the main lobes is inversely proportional to the total duration of the pulse package

$$\Delta t_0 \approx N_p T_p. \tag{7.4}$$

These relationships enable us to obtain an important connection between some of the analyzer parameters.

Thus, $\omega_1/\Delta\omega_1 = N_{p1}$ pulses will be required if it is required to determine the lower frequency component of the spectrum ω_1 with passband $\Delta\omega_1$. A time $t = N_{p1}T_1$, or $t = N_{p1}(2\pi)/\omega_1$, is required to form N_{p1} pulses. Then, in order to separate the higher frequency component of this band, ω_h, in time t, it is necessary to have N_{p2} pulses; $N_{p2} = t/t_h$. The passband for the higher frequency, ω_h, will equal $\Delta\omega_h = \omega_h/N_{p2}$, or $\Delta\omega_h = \omega_h T_h/t = \omega_1/N_{p1}$ /44

$$\Delta\omega_h = \Delta\omega_1. \tag{7.5}$$

Thus, the passband for all frequencies in this spectrum is the same, given constancy of time for analysis of the frequency components of the spectrum.

If constancy of number of pulses analyzed is used as the base, we obtain a change in passband for different frequencies for the entire band. Thus

$$\Delta\omega_1 = \frac{\omega_1}{N_p},$$

and

$$\Delta\omega_h = \frac{\omega_h}{N_p}.$$

Then

$$\Delta\omega_1 = \frac{\omega_1}{\omega_h}\Delta\omega_h, \quad \text{or} \quad \frac{\Delta\omega_1}{\Delta\omega_h} = \frac{\omega_1}{\omega_h}. \tag{7.6}$$

So we see that broadening the filter passband at the higher frequency of the band will be directly proportional to the ratio of the two extreme frequencies in this band.

The above can be used as the basis for determining the connection between the passband and the rate of change in the frequency of the internal oscillator: $\gamma = (f_h - f_1)/T_a$, where γ is the rate of change in the frequency of the internal oscillator. But the entire band of frequencies $f_h - f_1$ can be scanned with discreteness Δf; that is, $f_h - f_1 = n\Delta f$. This requires n switchings. These n switchings are required to provide the time to analyze the entire frequency spectrum, $T_a = nt_a$, where t_a is time for analysis of an individual band, Δf. The result is $\gamma = \Delta f/t_a$.

Moreover, we know that analysis time, t_0, depends on the resonant frequency and on the number of pulses passed by the internal oscillator. If we say that time t_a is assigned to each isolated frequency band, Δf, for any resonant frequency, then $t_a = 1/\Delta f$,

$$\gamma = \Delta f^2. \tag{7.7}$$

But if we say that the constant is the number of pulses passed by the internal oscillator, we obtain $t_a = N_p/f_r$

$$\gamma = f_r^2/N_p^2, \tag{7.8}$$

where

f_r is the resonant frequency.

Thus, use of a dischargable filter has made it possible to overcome the drawbacks in step-by-step analyzers associated with the dynamic resonance curve. We obtained the maximum possible rate of change in frequency. /45

8. Formation of the Analyzer's Square-Wave Characteristic Curve

An analyzer operating in the dischargeable filter mode has a conventional bell-shaped resonance characteristic curve. The filter reacts to the adjacent frequencies not contained in the passband because the curve has flat tails. The flat tails of the curve prevent making the determination of the frequency of the isolated component with the necessary accuracy.

If the resonant frequencies of the filters are separated so as to be free of adjacent signals, we can have the case when some of the frequency components of the input signal will not appear in any of the passbands of the isolated frequencies. In the ideal case we should have a Π-shaped curve for all resonant frequencies which touch each other with their passbands, forming the overall passband for the device. The square-wave resonance curve can be obtained from the individual curves for narrow-band filters. They can be formed as follows in the analyzer. The analyzer is tuned in turn to the individual frequencies, $\omega_1 \ldots \omega_m$. The analyzer's output signal is integrated. The filters and integrater are discharged when frequency ω_m is reached. The mean value for the band of frequencies in question is formed at the analyzer output.

If the passband for the individual filter equals $\Delta\omega$, the overall passband for the analyzer can be determined as

$$\Delta\Omega_\Sigma \approx \sum_1^m \Delta\omega_m = m\,\Delta\omega_m . \qquad (8.1)$$ /46

When constructing a square-wave curve using a dischargeable filter, consideration must be given to the fact that any resonant frequency in this band will be isolated in time Δt with period $m\Delta t$, but then the integral value of the output amplitude of the filter will equal $a = A(\Delta t/m\Delta t)$, where A is the amplitude value of the input frequency. If the filter curve is bell-shaped, it will be necessary to double the time this frequency is acting in order to form the filter's output signal from frequency ω_{12}, which is between the two resonant frequencies ω_1 and ω_2. So, when frequency ω_1 is isolated, frequency ω_{12} will pass at the output. The situation is similar for the isolation of resonant frequency ω_2 (Figure 16).

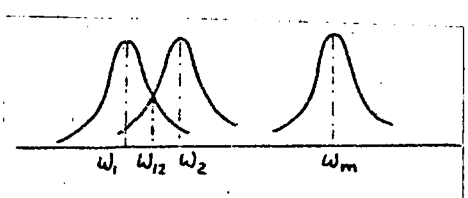

Figure 16.

Signal b_1A_{12} will be present at the output if the filter is tuned to frequency ω_1 because frequency ω_{12} will be acting. Here b is the attenuation factor for the bell-shaped curve, and A is the amplitude of the input signals. We obtain b_2A_{12} for frequency ω_2.

The following must prevail in order to obtain a flat peak on the resonance curve

$$A_1 = A_2 = (b_1 + b_2)\,A_{12}. \qquad (8.2)$$

We can set $b_1 = b_2$ when ω_{12} is located symmetrically between the frequencies and when there is identity between the resonance curves (we shall not consider the case when a narrow-band curve with high Q is used to isolate ω_1, and one with lesser Q is used to isolate ω_2). /47

Since $A_1 = A_2 = 2bA_{12}$, $b_1 = b_2 = 0.5$. In this case the resonance curve resultant is practically flat.

Since $\Delta\omega_{RC} = 3/\tau$ for the resonance curve for an RC filter at the 0.5 level, resonant frequencies ω_1 and ω_2 should be found at this same frequency distance.

Then the filter passband at the 0.7 level, fixed by frequencies ω_1 and ω_2 will consist of

$$\frac{\Delta\omega_1}{2} + \Delta\omega_{12} + \frac{\Delta\omega_2}{2} = \frac{5}{\tau}. \tag{8.3}$$

If the passband is formed from a large number of individual, identical, curves, the general equation will be in the form

$$\Delta\Omega_{0.7} = \frac{2+3(m-1)}{\tau} = \frac{3m-1}{\tau}. \tag{8.4}$$

For determination of the quality of rectangularity of a Π-shaped curve at two levels - 0.1 and 0.01. The bell-shaped curve has band $\Delta\omega_{0.1} \approx 20/\tau$ at the 0.1 level, and $\Delta\omega_{0.01} \approx 200/\tau$ at the 0.01 level. We obtain the following passbands, respectively, for the Π-shaped curve at these levels

$$\Delta\Omega_{0.1} = \frac{3m+17}{\tau}, \quad \Delta\Omega_{0.01} = \frac{3m+197}{\tau}. \tag{8.5}$$

In this case the rectangularity of the curve will equal

$$\Pi_{0.1} = \frac{\Delta\Omega_{0.1}}{\Delta\Omega_{0.7}} = \frac{3m+17}{\tau} \cdot \frac{\tau}{3m-1} = \frac{3m+17}{3m-1}. \tag{8.6}$$

$$\Pi_{0.01} = \frac{3m+197}{3m-1}. \tag{8.7}$$

We can simplify these expressions for $m > 3$

$$\Pi_{0.01} = \frac{3m+17}{3m} \approx 1 + \frac{6}{m}. \tag{8.8}$$ /48

$$\Pi_{0.01} = 1 + \frac{66}{m}. \tag{8.9}$$

A rectangularity of $\Pi_{0.1} \approx 2\text{-}3$ can be taken as practically acceptable for a great many tasks, as work with transmission gates has shown. The expression obtained for $\Pi_{0.1}$ can be used to obtain the calculated rectangularity, even for values of $m = 4$, a completely acceptable magnitude.

Determination of rectangularity $\Pi_{0.01}$ equates primarily to those devices with more rigid discrimination specifications. Values of $\Pi_{0.01} \approx 5$ were obtained here. It is necessary to have m = 16 in order to obtain a similar rectangularity in our case.

9. Determination of Amplitude-Frequency and Phase-Frequency Spectra of a Signal in Question Using an Electronic Computer

The foregoing was concerned primarily with consideration of an analog method of obtaining the amplitude-frequency and phase-frequency spectra of a signal in question. But we can, in addition to this method, build a coherent analyzer with a digital filtering method. The input signal must be quantized and converted into a binary code for this purpose. The input signal can be replaced by a set of numbers, Figure 17a

$$x(t) \rightarrow x_1, x_2, x_3, \ldots\ldots\ldots x_n$$

The input signal conversion frequency is f_n, and the conversion period is τ_n.

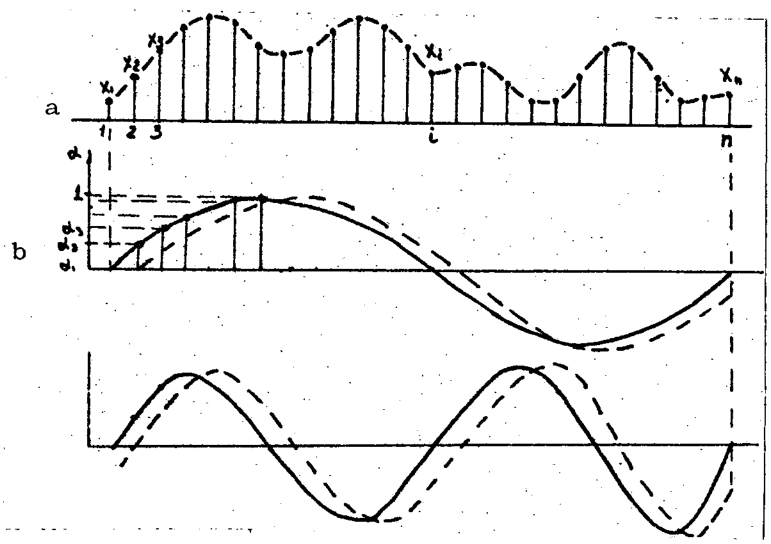

Figure 17.

Let us determine the amplitude and phase of the frequency components of the function x(t). The constant component of this function equals

$$A_0 = \frac{1}{n} \sum_{n=1}^{n} x_n , \tag{9.1}$$

where

n is the total number of measurements.

The amplitude of the signal with the lowest frequency can be determined if all measurements of signal x(t) are broken down into two large groups. Each result of a measurement in a group is multiplied by a definite factor determined by the sine function, Figure 17b. /49

Then the amplitude of the first harmonic can be determined as

$$A_{11} = \frac{1}{n}\sum_{n=1}^{i} \alpha_n X_n - \frac{1}{n}\sum_{i+1}^{n} \alpha_n X_n . \qquad (9.2)$$

The coefficients of the sine function must be shifted one measurement to the right in order to determine the phase of this frequency component. We obtain the following new value for the amplitude of the first harmonic.

$$A_{12} = \frac{1}{n}\sum_{n=2}^{i+1} \alpha_n X_n - \frac{1}{n}\sum_{i+2}^{i+K} \alpha_n X_n . \qquad (9.3)$$

Thus, by moving α one whole set of measurements, we obtain n values for the amplitudes of the first harmonics of the function x(t). Selected from all amplitude values is the maximum amplitude of the first harmonic, A_{1m}, and the time of appearance of this maximum with respect to the number of discrete shifts in α is fixed, that is

$$A_{1m} = \frac{1}{n}\sum_{m}^{m+i} \alpha_n X_n - \frac{1}{n}\sum_{m+i+1}^{m+K} \alpha_n X_n . \qquad (9.4)$$

The magnitude of the phase of this harmonic can be determined with accuracy

$$\Delta\varphi = \frac{2\pi}{n} . \qquad (9.5)$$

The value of the phase of the first harmonic is equal to

$$\varphi_i = \frac{2\pi}{n} \cdot m . \qquad (9.6)$$

We can use a similar method to determine the amplitude and phase of the next harmonic, with the only difference being that the number of measurements during the period of activity of the second harmonic will be n - 1.

If the frequency of the first harmonic is equal to

$$f_1 = \frac{1}{n\tau_n} , \qquad (9.7)$$

the second harmonic will have the value /51

$$f_2 = \frac{1}{(n-1)\tau_n} \qquad (9.8)$$

the third harmonic

$$f_3 = \frac{1}{(n-2)\tau_n}. \tag{9.9}$$

The maximum frequency in the spectrum will be

$$f_m = \frac{1}{2\tau_n}. \tag{9.10}$$

If the spacing of α for isolating the frequency component is considered to be $f_s = 2f_1$, it will be seen that the number of measurements involved in the determination of the harmonic's amplitude and phase will be reduced. But this results in a reduction in accuracy in determining the phase of the frequency component of the input signal. Thus, the accuracy in determining the phase of the second harmonic is equal to

$$\Delta\varphi_2 = \frac{2\pi}{n-1}, \tag{9.11}$$

and the accuracy in determining the phase of the third harmonic is equal to

$$\Delta\varphi_3 = \frac{2\pi}{n-2}. \tag{9.12}$$

The phase of the harmonic with maximum possible frequency resolution will be determined with accuracy

$$\Delta\varphi_m = \frac{2\pi}{2} = \pi. \tag{9.13}$$

Thus, accuracy in phase determination decreases with increase in frequency. Consequently, if we must have the upper frequency of the input signal spectrum, f_u, with the accuracy that for determination of the phase of this harmonic, $\Delta\varphi_u$, the frequency of quantization of the input signal will equal

$$f_n = \frac{2\pi}{\Delta\varphi_u} f_u. \tag{9.14}$$

This method of converting an input signal can be widely used in remote data processing systems to reduce the time required to transmit telemetry data. /52

However, the number of measurements made of a signal in question can be limited because of the limited volume of the memory of these systems. In this case the input signal must be processed in steps. Signal x(t), for example, can be processed for a time segment t_1 to t_n, where t_1 can be zero, to form two characteristics

$$x(t_1 - t_n) \to A_1(\omega),\ \varphi_1(\omega), \tag{9.15}$$

and for the time segment from t_n to t_{zn}

$$x(t_n - t_{zn}) \to A_2(\omega),\ \varphi_2(\omega)\quad \text{etc.} \tag{9.16}$$

The result is that we obtain a set of spectra that equate to different moments in time for the signal in question. Further, these spectra can be combined into one when we consider that spectra $A_2(\omega)$, $\varphi_2(\omega)$ are shifted relative to the origin of the reading, t_1, by phase shift $\varphi_2 = \omega t_1$.

In this case it must be considered that

$$\bar{\varphi}_2(\omega) = \varphi_2(\omega) + \omega t_1. \tag{9.17}$$

The resultant amplitude and phase of each frequency component can be found by using the known formulas

$$A_{12} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\,(\varphi_1 + \bar{\varphi}_2)}, \tag{9.18}$$

$$\boxed{\operatorname{tg} \varphi_{12} = \frac{A_1 \sin \varphi_1 + A_2 \sin \bar{\varphi}_2}{A_1 \cos \varphi_1 + A_2 \cos \bar{\varphi}_2}} \tag{9.19}$$

Similar conversions can be made with the other spectra as well. Thus

$$\begin{tikzcd}[column sep=tiny]
A_1 \arrow[dr] & & A_2 \arrow[dl] & A_3 \arrow[dr] & & A_4 \arrow[dl] & A_5 \arrow[dr] & & A_6 \arrow[dl] & A_7 \arrow[dr] & & A_8 \arrow[dl] \\
& A_{12} \arrow[drr] & & & A_{34} \arrow[dl] & & & A_{56} \arrow[drr] & & & A_{78} \arrow[dl] & \\
& & & A_{23} \arrow[drr] & & & & & & A_{67} \arrow[dllll] & & \\
& & & & & A_{36} & & & & & &
\end{tikzcd} \tag{9.20}$$

The result is that we obtain spectra $A_{36}(\omega)$ and $\varphi_{36}(\omega)$, which characterize the time function for time $T_1 = t_{3n} - t_1$.

As will be seen from the foregoing, the signal in question can be resolved into frequency components if it has a limited band of frequencies, $\Delta F = f_m - f_1$, and is limited to time T_1. /53

The maximum time signal $x(t)$ will be active will be determined by the beat frequency for frequencies f_1 and f_2

$$\Delta f = f_2 - f_1 \tag{9.21}$$

or

$$\Delta f = \frac{1}{(n-1)\tau_n} - \frac{1}{n\tau_n} = \frac{1}{\tau_n \cdot n(n-1)}. \tag{9.22}$$

Considering that $n \gg 1$, we obtain

$$\Delta f = \frac{1}{n^2 \tau_n}. \tag{9.23}$$

So the time the input signal will be active will equal

$$T_1 = \frac{1}{\Delta f} = n^2 \tau_n. \tag{9.24}$$

Thus, we must have longer realization of the input signal in order to increase the time it is active. This means that we must decrease the frequency of the spectrum to the minimum possible.

In order to determine the frequency of the spectrum $f_{low} < f_1$ using the same number of measurements that we used in the preceding case, we must conditionally increase the length of the realization, Figure 18a. We can take it that measurements $x_1, x_2, \dots x_n$ are within the initial sections of the time segment. All measurements for time $t_k > t_n$ can be equal to some constant, or, in the frequency case, to zero. For simplicity, we will take it that $x_{n+1}, x_{n+2}, \dots x_k = 0$. Then, as in the first case, the amplitude of the harmonic with the low frequency, $f_{1\ low}$, will be determined as

$$A_{1\ low} = \frac{1}{K}\sum_{K=1}^{j} \alpha_K X_K - \frac{1}{K}\sum_{j+1}^{K} \alpha_K X_K. \tag{9.25}$$

but

$$\sum_{j+1}^{K} \alpha_K X_K = 0, \; a \sum_{K=1}^{j} \alpha_K X_K = \sum_{K=1}^{n} \alpha_K X_K + \sum_{n+1}^{j} \alpha_K X_K.$$

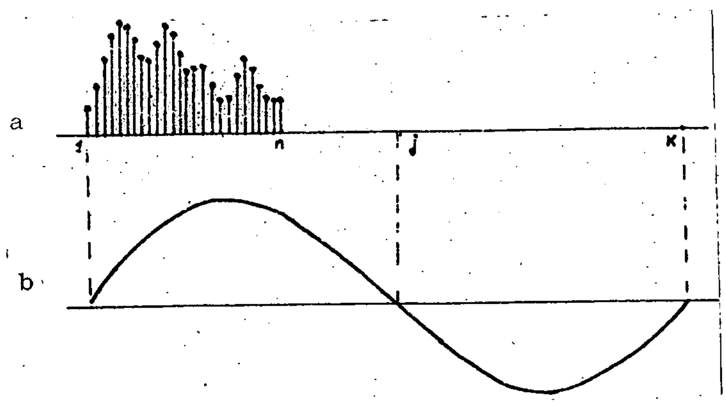

Figure 18.

Here too $\sum_{n+1}^{i} d_k x_k = 0$, then /54

$$A_{1\ low} = \frac{1}{k} \sum_{k=1}^{n} d_k x_k. \tag{9.26}$$

The amplitude of the harmonic with the other phase can be determined as in the preceding case. All succeeding harmonics of the input signal can be determined similarly.

The result is a set of spectra

$$x(t_1 - t_k) \rightarrow \begin{cases} A_{1\ low}(\omega),\ \varphi_{1\ low}(\omega) \\ A_{2\ low}(\omega),\ \varphi_{2\ low}(\omega) \\ \dots\dots\dots\dots \\ A_{i\ low}(\omega),\ \varphi_{i\ low}(\omega) \end{cases} \tag{9.27}$$

These spectra can be converted into a single spectrum by using Eqs. (9.18) and (9.19). Here it should be pointed out that summing the harmonic components belonging to the different spectra can be accomplished without considering the time shift because all these spectra belong to one time realization.

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